GOVT. DEGREE COLLEGE FOR MEN, SRIKAKULAM

DEPARTMENT OF MATHEMATICS

STUDENT SEMINARS
2021-22

Date: 22-02-202

S.No.	Name of the Student	Hallticket No.	Group	Topic	Signature
1	T. Madavi	1900140052	MECS	Basic Extension Theorem	T. Madavi
2	P. Leelavathi	1900140032	MECS	1.S.T (1,2,1), (2,1,1) & (1,1,2) form a basis for $\mathbb{R}^3(\mathbb{R})$ 2.Determine $\left\{\begin{bmatrix} a & b & c \\ b+c & 0 & a+b \end{bmatrix}, a,b \in R\right\}$ is a subspace of $\mathbb{R}_{2\times 3}$	P. Leelavathi
3	L. Sai Durga Prasad	1900142030	MPCS	Find the Transformation Matrix from the basis (e_1, e_2, e_3) to the Basis (1,2,0), (2,-1,3) & (1,1,1)	Lolla. Saidurga prasad.
4	P. Anitha	1900142041	MPCS	If $W1$ & $W2$ are subspaces of a finite dimensional vector space V(f) then $dim(W1 + W2) = dim(W1) + dim(W2) - dim(W1 \cap W2)$	P. Anitha
5	S. Koteswara Rao	19001410044	MPC	Find the characteristic roots & the corresponding characteristic vectors of the Matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	Seepana. koteswara Rao
6	L. Ramya	1900142029	MPCS	State and Prove Cayley-Hamilton Theorem	Loddi. Ramya
7	L. Krishna Rao	1900140024	MECS	Explain Cauchy-Schwarz & Bessel's Inequality	[illegible]
8	A. Hitesh Reddy	1900140005	MECS	Linear Sum & Sub Space	A. Hitesh Reddy
9	P. Rakesh	1900140032	MECS	Verify C-H Theorem for the Matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$	Pudi. Rakesh
10	P. Anil Kumar	1900140043	MECS	State and Prove Stoke's Theorem	P. Anil kumar
11	T. Mohini Kumar	1900142054	MPCS	State & Prove Green's Theorem	T. Mohinikumari
12	K. Ananth Rao	1900143020	MPE	State & Prove Gauss Divergence Theorem	K. Ananth Rao

21) 2.3.3

Show that (1,2,1), (2,1,1) and (1,1,2) form a basis for R^3 (R)

Space for solution:

let $a(1,2,1)+b(2,1,1)+c(1,1,2)=0$ for $a,b,c \in R$

$(a,2a,a)+(2b,b,b)+(c,c,2)=0$

$\Rightarrow (a+2b+c);(2a+b+c),(a+b+2)=0,(0,0,0)$

$a+2b+c=0$, $2a+b+c=0$, $a+b+2=0$

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$x=(x_1,x_2,x_3)=a(1,2,1)+b(2,1,1)+c(1,1,2)$
$=(a+2b+c),(2a+b+c$

$\Rightarrow 1(2-1)-2(4-1)+1(2-1)=1-6+1=-4\neq 0$

$\therefore a=b=c=0$ the given set is linearly indepedent

The arguementad matrix (i)

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} \xrightarrow[R_3\to R_3-R_1]{R_2\to R_2-2R} \left[\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 1 & -3 & -1 & x_2-2x_1 \\ 0 & -1 & 1 & x_3-x_1 \end{array}\right] \xrightarrow{R_3\to 3R_3-R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 0 & -3 & -1 & x_2-2x \\ 0 & 0 & 4 & x_1-x_2+3x_3 \end{array}\right]$$

$a+2b+c=x_1$
$-3b-c=x_2-2x_1$
$4c=-x_1-x_2+3x_1$
$c=\frac{-x_1-x_2+3x_3}{4}$

$-3b=x_2-2x_1+c$
$=x_2-2x_1+(-x_1-x_2+3x_3)$
$=\frac{4x_2-8x_1-x_1-x_2+3x_3}{4}$
$=\frac{-9x_1+3x_2+3x_3}{4}$
$b=\frac{+3x_1-x_2-x_3}{4}$

$a+2b+c=x_1$
$a=x_1-2b+c$
$=x_1+2(8x_1-x_2-x_3)$
$+\frac{x_1-x_2-3x_3}{4}$
$=\frac{9x_1-3x_2+x_3}{4}$

$\therefore x=\frac{9x_1-3x_2+x_3}{4}(1,2,1)+\frac{3x_1-x_2-x_3}{4}(2,1,1)+\frac{(-x_1,x_2+3x_3)}{4}(1,1,2)$

The given set is linearly indepent and spans R3 and hence is a basis of R3

Signature: P. Leelavathi

(22) 3.3.1

find the transformation matrix from the basis $\{e_1, e_2, e_3\}$ to the basis $\{(1,2,0), (2,-1,3), (1,1,1)\}$.

space for solution:

In $V_3(R)$,

$e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

let us express the elements of Basis as linear combinations of e_1, e_2, e_3,

$(1,2,0) = a(1,0,0) + b(0,1,0) + c(0,0,1)$

$= (a,0,0) + (0,b,0) + (0,0,c)$

$= (a,b,c)$

$\Rightarrow a=1, b=2, c=0$ (equating the corresponding coordinates)

$\therefore (1,2,0) = 1(1,0,0) + 2(0,1,0) + 0(0,0,1)$

similarly $(2,-1,3) = 2(1,0,0) + (-1)(0,1,0) + 3(0,0,1)$

$(1,1,1) = 1(1,0,0) + 1(0,1,0) + 1(0,0,1)$

The transformation Matrix

$$P = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

(23) 3.3.2

(a) Determine $\left\{ \begin{bmatrix} a & b & c \\ b+c & 0 & a+b \end{bmatrix} : a,b \in R \right\}$ is a subspace of $R_{2\times 3}$?

(b) Determine if $\{x_1 = (1,-1,0), x_2 = (0,2,1), x_3 = (2,4,3)\}$ is a subspace of R^3?

space for solution:

(b) first let us verify the closure axiom of addition

$(1,-1,0) + (0,2,1) = (1,1,1) \notin W$

$(0,2,1) + (2,4,3) = (2,6,4) \notin W$

Here '+' is not closed in 'W' Hence W is not a subspace.

5.2 Practical Problems

1. Determine the characteristic roots of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$

Space for solution:

The characteristic equation $A = |A - \lambda I| = 0$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} -\lambda & 1 & 2 \\ 2 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{bmatrix} = 0$$

$\Rightarrow -\lambda(\lambda^2 - 1) + 1(-\lambda + 2) + 2(-1 + 2\lambda) = 0$

$\Rightarrow -\lambda^3 + \lambda - \lambda + 2 - 2 + 4\lambda = 0$

$\Rightarrow -\lambda^3 + 4\lambda = 0$

$\Rightarrow \lambda(4 - \lambda^2) = 0$

$\Rightarrow \lambda(2 + \lambda)(2 - \lambda) = 0$

$\lambda = 0, \lambda = -2, \lambda = 2$

$\lambda = -2, 0, 2$

2. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

Solution:

$$|A - \lambda I| = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 & 1 \\ 0 & 1-\lambda & -1 \\ 3 & -1 & 1-\lambda \end{bmatrix}$$

$\Rightarrow (1-\lambda)\{(1-\lambda)^2 - 1\} - 2(0+3) + 1(0 - 3 + 3\lambda)$

$= (1-\lambda)(\cancel{1} + \lambda^2 - 2\lambda - \cancel{1}) - 2 \times 3 + 1(-3 + 3\lambda)$

$= 1(\lambda^2 - 2\lambda) - \lambda(\lambda^2 - 2\lambda) - 6 - 3 + 3\lambda$

$= \lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2 - 6 - 3 + 3\lambda$

$= -\lambda^3 + 3\lambda^2 + \lambda - 9$

The characteristic equation is given by $-\lambda^3+3\lambda^2+\lambda-9=0$

By Cayley - Hamilton theorem, $-A^3+3A^2+A-9I=0$ — (1)

Now we verify equation — (1)

$$A^2 = \begin{bmatrix}1&2&1\\0&1&-1\\3&-1&-1\end{bmatrix}\begin{bmatrix}1&2&1\\0&1&-1\\3&-1&1\end{bmatrix} = \begin{bmatrix}1+0+3&2+2-1&1-2+1\\0+0-3&0+1+1&0-1-1\\4+0+3&6-1-1&3+1+1\end{bmatrix} = \begin{bmatrix}4&3&0\\-3&2&-2\\6&4&5\end{bmatrix}$$

$$A^3 = \begin{bmatrix}4&3&0\\-3&2&-2\\6&4&5\end{bmatrix}\begin{bmatrix}1&2&1\\0&1&-1\\3&-1&1\end{bmatrix} = \begin{bmatrix}4-6+6&3+4+4&0-4+5\\0-3-6&0+2-4&0-2-5\\12+3+6&9-2+4&0+2+5\end{bmatrix} = \begin{bmatrix}4&11&1\\-9&-2&-7\\21&11&7\end{bmatrix}$$

Now, $-A^3+3A^2+A-9I$

$$= -\begin{bmatrix}4&11&1\\-9&-2&-7\\21&11&7\end{bmatrix} + 3\begin{bmatrix}4&3&0\\-3&2&-2\\6&4&5\end{bmatrix} + \begin{bmatrix}1&2&1\\0&1&-1\\3&-1&1\end{bmatrix} - 9\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}4&-11&-1\\9&2&7\\-21&11&-7\end{bmatrix} + \begin{bmatrix}12&9&0\\-9&6&-6\\18&12&15\end{bmatrix} + \begin{bmatrix}1&2&1\\0&1&-1\\3&-1&1\end{bmatrix} - \begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$$

$$= \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} = 0$$

Hence Calyigy - Hamilton theorem is verified.

(25) 6.3.1

Find the rank of the matrix A by reducing into echelon form where $A = \begin{bmatrix}2&-1&3&4\\0&3&4&1\\2&3&7&5\\2&5&11&6\end{bmatrix}$

Solution:

$$A = \begin{bmatrix}2&-1&3&4\\0&3&4&1\\2&3&7&5\\2&5&11&6\end{bmatrix}$$

$R_3 \to R_3 - R_1$; $R_4 \to R_4 - R_1$

$$A = \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 6 & 6 & 8 & 2 \end{vmatrix}$$

$R_4 \to R_4 - 2R_2$

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of A = The number of nonzero rows in echelon form = 3

6.3.2

Find the rank of the Matrix A by reducing into normal form where $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

Solution:

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$$A = \begin{vmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$C_3 \to C_3 - 2C_2$, $C_4 \to C_4 - 5C_2$

$$A = \begin{vmatrix} 1 & 1 & 1 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$C_3 \to C_3 + C_1$, $C_4 \to C_4 + 6C_1$

$$A = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 2 \begin{bmatrix} I_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rant of given Matri - 2

$= 0 =$

(27) 6.3.3

Solve the system of equations

$x + 2y + 3z = 0$

$2x + 3y + 4z = 0$

$7x + 13y + 19z = 0$

Solution:

The matrin equation of given system $AX = 0$ is given by

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 7 & 13 & 19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 7 & 13 & 19 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3\ 7R_1$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

The rank of coefficient matrix is 2

Now the number of unknowns is 3

$\therefore$ The given system possess $3-2=1$

linearly independent solution, we assign arbitrary values $n-r=3-2$

The given system can be written as

$$x+2y+3z=0$$
$$-y-2z=0$$
$$\therefore y=-2z, \ x=-z$$

Put $z=c$, Then $y=-2c$, $x=-c$

Hence $x=-c$, $y=-2c$, $z=c$ will constitute a solution for the given system of equasions.

7.3.1

Obtain the matrix for the quadratic form $x^2+2y^2+3z^2+4xy+5yz+6zx$?

Solution:

Given equastion $x^2+2y^2+3z^2+4xy+5yz+6zx$

it can be written as.

$$x^2+2xy+3xz+2yx+2y^2+\frac{5}{2}yz+3zx+\frac{5}{2}zy+3z^2$$

$\therefore$ If A is the quadratic form, then

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 2 & 5/2\\ 3 & 5/2 & 3\end{bmatrix}$$

which is symmetric matrix of order 3.

7.4.2

prove that the quadratic form

$$6x^2+4y^2+5z^2-8yz+20zx-4xy$$

in three variables is positive definite

Solution:

Given quadratic form

$6x^2+49y^2+51z^2-4xy-82yz+20zx$

The Matrix A of given quadratic form is

$$A = \begin{bmatrix} 6 & -2 & 10 \\ -2 & 49 & 41 \\ 10 & -41 & 51 \end{bmatrix}$$

The leading principle minors of A are

$A_1 = 6$

$A_2 = \begin{vmatrix} 6 & -2 \\ -2 & 49 \end{vmatrix} = 294-4 = 290$

$A_2 = \begin{vmatrix} 6 & -2 & 10 \\ -2 & 49 & -41 \\ 10 & -41 & 51 \end{vmatrix} = 6(2599-881)$

$= 6(1718)$

$= 11,308$

since the leading principle minors of A and all positive we have the given quasions form is positive definitiy.

$=0=$

(30) 7.4.3

Show that for the vectors $x=(x_1,x_2)$ and $y=(y_1,y_2)$ form R^2 the following defines an inner product on R^2

$(x,y) = x_1y_1 - x_2y_1 - x_1y_2 + 2x_2y_2.$

Solution:- we will show that all the positive of an inner product hold good.

1. Symmetry:-

$(\beta,\alpha) = y_1x_1 - y_2x_1 - y_1x_2 + 2x_2y_2$

$= x_1y_1 - x_2y_1 - x_1y_2 + 2x_2y_2$

$= (\alpha,\beta)$

Linearity

let $a, b \in R$ then $ax+by = a(x_1, x_2) + b(y_1, y_2)$

$= (ax_1, ax_2) + (by_1, by_2)$

$= (ax_1 + by_1, ax_2 + by_2)$

let $z = (z_1, z_2) V_2(R)$

$(ax+by, z) = (ax_1+by_1)z_1 - (ax_2 by_2)z_1 - (ax_1+by_1)z_2 + 2(ax_2+by_2)z_2$

$= ax_1z_1 - ax_2z_1 - ax_1z_2 + 2ax_2z_2 + by_1z_1 - by_1z_1 - by_2z_1$

$- by_1z_1 - by_1z_2 + 2by_2z_2$

$= a(x_1z_1 - x_2z_1 - x_1z_2 + 4x_2z_2) + b(y_1z_1 - y_2z_1 - y_1z_2 + 2y_2z_2)$

$= a(x, z) + b(y, z)$

non - negativity

we have $(x, x) = x_1x_1 - x_2x_1 - x_1x_2 + 2x_2x_2$

$= x_1^2 - 2x_1x_1 + 2x_2^2$

$= (x_1 - x_2)^2 + x_2^2$ which is the same of them.

non - negative real numbers

$(x, x) \geq 0$

$(x, x) = 0 \Rightarrow (x_1 - x_2)^2 + x_2^2 = 0$

$(x_1 - x_2)^2 = 0, x_2^2 = 0$

$x_1 = x_2 = 0, x_2 = 0$

$= 0$

$x_1 = 0, x_2 = 0$ $\therefore$ the given product is an inner product

8.3.2

Prove that the two vectors u and v in a real inner product space are orthogonal if and only if $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

Solution :-

$\|u+v\|^2 = \|u\|^2 + \|v\|^2 \Leftrightarrow (u+v, u+v) = \|u\|^2 + \|v\|^2$

$\Rightarrow (u, v) + (u, v) + (v, u) + (v, v) = \|u\|^2 + \|v\|^2$

$\Rightarrow \|u\|^2 + (u, v) + (\bar{u}, v) + \|v\|^2 = \|u\|^2 + \|v\|^2$

$\Rightarrow (u, v) + (u, v) = 0$ $\therefore (u, v$ is real. $(\bar{u}, v) = (u, v))$

T. MADAVI, Hall Ticket NO: 1900140052 Date 22-02-2022

Theorem: Basic Extension Theorem:-

1. Explain Basic & Daimens, Basic externs theorem.

Let V(f) be a finite dimentional vector space. let $s = \{a_1, a_2 --- a_r\}$ be a linearly Independent Subset of V. Then "S" itself is a basic of V or there exists a basic of V which contains S. (i.e) S can be Extended to from a basic of V.

Proof:-

V(f) is a finite dimensional V(S)

$S = \{a_1, a_2 --- a_r\}$ is linearly independent. if S Span V, then S itself is a basis. if do not span V, Then there Exists a vector a_{r+1} in V whic Can not be written as a linear Combination of $a_1, a_2 \cdots a_r$.

let $S_1 = \{a_1, a_2 --- a_r, a_{r+1}\}$. we observe that S_1 is linearly independent.

let $a_1 a_2 + a_2 a_2 + \cdots + a_r a_r + a_{r+1} = 0 \rightarrow (1)$

where $a_1, a_2, \cdots a_{r+1} \in F$.

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Signature :

if $a_{r+1} \neq 0$ then there exists $a_{r+1}^{-1} \in F$. Such that

$a_{r+1} a_{r+1}^{-1} = a_{r+1}^{-1} a_{r+1} = 1$

Now multiplying both sides of (1) by a_{r+1}^{-1} we can see that a_{r+1} is a linear combination of $a_1, a_2 \dots a_r$, which is false.

Hence $a_{r+1} = 0$

$\therefore a_1 a_1 + a_2 a_2 + \dots + a_r a_r = \bar{0}$

Since $\{a_1, a_2, \dots a_r\}$ is linearly Independent a_1

$a_1 = a_2 = \dots = a_r = 0$

Hence S_1 is linearly Independent

if S_1 span V then S_1 is a basic of V and contain S.

if S_1 do not span V then there Exists a vector a_{r+2} in V. which can not be written as a linear combination of the Element of S_1.

let $S_2 = \{a_1, a_2 \dots, a_r . a_{r+1}, a_{r+2}\}$.

Then as above we can show that S_2 is linearly Independent. if S_2 span V then S_2 is a basic of V and S_2 contains S. if $L(S_2) \neq V$

Subhodaya

then we repeat the above procedure. This procedure continues infinitely then we get an infinite number of linearly independent vectors in a finite dimensional vector space which is false [Because in a finite dimensional vector space the number of spanning vectors is finite. and hence the number linearly independent vectors is also finite]. Hence at some strange. we get a set S_k which is linear independent and span v. Hence S_1 is the basics of V and also S_k contains S.

NOTE :-

if v(f) is a finite dimentional vector space then any every basics of V is finite.

Signature :- T. madani

4. If W_1 & W_2 are Subspaces of a finite dimensional vector space $V(F)$ then.

$$\dim(W_1+W_2) = \dim(W_1) + \dim W_2 - \dim(W_1 \cap W_2)$$

Proof:

W_1, W_2, W_1+W_2; $W_1 \cap W_2$ are SubSpaces of the finite dimensional vector Space $V(F)$. Hence $W_1, W_2 + W_2 \cap W_2$ are also finite dimensional

let $S = \{r_1, r_2 --- r_k\}$ be a basic of $W_1 \cap W_2$ so that

$\dim(W_1 \cap W_2) = k$.

Now $W_1 \cap W_2 \subset W_1$, Hence S is linearly Independent in W_1. Hence by the basis Extension theorm, S can be extended to from a basis of W_1.

Let $S_1 = \{\alpha_1, \alpha_2, \alpha_3 --- \alpha_n, \gamma_1, \gamma_2 ---- \gamma_k\}$ be a basis of W_1

$\therefore \dim W_1 = n+k$

Similarly by the Same argument, S can be extended to form a basis of W_2.

Let $S_2 = \{\beta_1, \beta_2, --- \beta_m, \gamma_1, \gamma_2 --- \gamma_k\}$ be a basic of W_2

Hence $\dim W_2 = m+k$

Now $\dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

$= n+k+m+k-k$

$= n+m+k$.

We shall prove that $\dim(W_1+W_2)=n+m+k$.

for this we show that there exists a basic of W_1+W_2 which contains $n+m+k$ elements

let $S'=\{\alpha_1,\alpha_2,\dots\alpha_n,\beta_1,\beta_2,\dots\beta_m,\gamma_1,\gamma_2\dots\gamma_k\}$

we shall prove that S' is a basis of W_1+W_2

S' is linearly independent:-

let $a_1\alpha_1+a_2\alpha_2+\dots+a_n\alpha_n+b_1\beta_1+b_2\beta_2+\dots+$

$+b_m\beta_m+c_1\gamma_1+c_2\gamma_2+\dots+c_k\gamma_k=\bar{0} \longrightarrow ①$

for some $a_i^s, b_i^s, c_i^s \in F$

Now $b_1\beta_1+b_2\beta_2+\dots+b_m\beta_m$

$=-(a_1\alpha_1+a_2\alpha_2+\dots+a_n\alpha_n+c_1\gamma_1+c_2\gamma_2+\dots+c_k\gamma_k)$

$a_1\alpha_1+a_2\alpha_2+\dots+a_n\alpha_n+c_1\gamma_1+c_2\gamma_2+\dots+c_k\gamma_k\in W_1$

(Because it is a LC of the elements of W_1)

$\therefore b_1\beta_1+b_2\beta_2+\dots+b_m\beta_m\in W_1$

but $b_1\beta_1+b_2\beta_2+\dots+b_m\beta_m$

$=b_1\beta_1+b_2\beta_2+\dots+b_m\beta_m+0\cdot\gamma_1+0\cdot\gamma_2+\dots+0\,\gamma_k\in W_2$

Hence $b_1\beta_1+b_2\beta_2+\dots+b_m\beta_m\in W_1\cap W_2$

Since $\{\gamma_1,\gamma_2\dots\gamma_k\}$ is a basis of $W_1\cap W_2$ we can

Signature: P Anitha

write $b_1\beta_1 + b_2\beta_2 + \cdots + b_m\beta_m = d_1\gamma_1 + d_2\gamma_2 + \cdots + d_k\gamma_k$ for some $d_i^s \in F$

$\Rightarrow b_1\beta_1 + b_2\beta_2 + \cdots + b_m\beta_m - d_1\gamma_1 - d_2\gamma_2 - \cdots - d_k\gamma_k = \bar{0}$ for some $d_i^s \in F$

But $\{\beta_1, \beta_2, \ldots \beta_m, \gamma_1, \gamma_2 \ldots \gamma_k\}$ is a basic of W_2 & hence linearly independent

$\therefore b_1 = b_2 = \cdots b_m = d_1 = d_2 = \cdots = d_k = 0$

Hence by (1)

$a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n + c_1\gamma_1 + c_2\gamma_2 + \cdots + c_k\gamma_k = \bar{0}$

But $\{\alpha_1, \alpha_2 \ldots \alpha_n, \gamma_1, \gamma_2 \ldots \gamma_k\}$ is a basis of W_1 & hence linearly independent.

$\therefore a_1 = a_2 = \cdots = a_n = c_1 = c_2 = \cdots = c_k = 0$

$\therefore S'$ linearly independent.

S' span $W_1 + W_2$

let $\gamma \in W_1 + W_2$

we can write $\gamma = \alpha + \beta$ for some $\alpha \in W_1$ & $\beta \in W_2$

$\alpha \in W_1$ & S_1 is a basis of W_1, Hence we can write

$\alpha = \sum_{i=1}^{k} a_i\alpha_i + \sum_{i=1}^{k} b_i\gamma_i$ for some $a_i^s\, b_i^s \in F$

$\beta \in W_2$ & S_2 is a basis of W_2 then we can write

$\beta = \sum_{i=1}^{M} d_i\beta_i + \sum_{i=1}^{k} e_i\gamma_i$ for some of $d_i^s, e_i^s \in F$

write $b_1\beta_1 + b_2\beta_2 + \cdots + b_m\beta_m = d_1\gamma_1 + d_2\gamma_2 + \cdots + d_k\gamma_k$ for Some $d_i^s \in F$

$\Rightarrow b_1\beta_1 + b_2\beta_2 + \cdots + b_m\beta_m - d_1\gamma_1 - d_2\gamma_2 - \cdots - d_k\gamma_k = \bar{0}$ for

Some $d_i^s \in F$

But $\{\beta_1, \beta_2, \ldots \beta_m, \gamma_1, \gamma_2 \ldots \gamma_k\}$ is a basic of W_2 & hence linearly independent.

$\therefore b_1 = b_2 = \cdots b_m = d_1 = d_2 = \cdots = d_k = 0$

Hence by (1)

$a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n + c_1\gamma_1 + c_2\gamma_2 + \cdots + c_k\gamma_k = \bar{0}$

But $\{\alpha_1, \alpha_2 \ldots \alpha_n, \gamma_1, \gamma_2 \ldots \gamma_k\}$ is a basis of W_1 & hence linearly independent.

$\therefore a_1 = a_2 = \cdots = a_n = c_1 = c_2 = \cdots = c_k = 0$

$\therefore$ S' linearly independent.

S' span $W_1 + W_2$

let $\gamma \in W_1 + W_2$

we can write $\gamma = \alpha + \beta$ for some $\alpha \in W_1$ & $\beta \in W_2$

$\alpha \in W_1$ & S_1 is a basis of W_1, Hence we can write.

$$\alpha = \sum_{i=1}^{k} a_i\alpha_i + \sum_{i=1}^{k} b_i\gamma_i \text{ for Some } a_i^s\, b_i^s \in F$$

$\beta \in W_2$ & S_2 is a basis of W_2 then we can write.

$$\beta = \sum_{i=1}^{M} d_i\beta_i + \sum_{i=1}^{k} e_i\gamma_i \text{ for Some of } d_i^s, e_i^s \in F$$

Hence $\gamma = \alpha + \beta = \sum_{i=1}^{n} \alpha_i \alpha_i + \sum_{i=1}^{m} \alpha_i \beta_1 + \sum_{i=1}^{k} (b_i + e_i) \gamma_1$

which is a ~~linear~~ combination of the elements of S'.

Hence S' Span $W_1 + W_2$

$\therefore S'$ is a based of $W_1 + W_2$

$\therefore \dim(W_1 + W_2) = n + m + K$

$= \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$

5. Find the characteristic roots & the Corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Aim:- To find the characteristic roots & the corresponding characteristic vector of the given matrix.

Formula:-

(i) If A is square matrix of order n & I is the $n \times n$ Unit matrix the $|A - \lambda I| = 0$ where λ is a Scalar, is Called the Characteristic Equation of A & the roots of the characteristic eqn are the characteristic roots of A.

(ii) let A be a Square matrix then a non zero vector x is called a characteristic vector of A, if there Exists a Scalar λ Such that $Ax = \lambda x$.

Procedure:-

Given that $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Then the Characteristic eq^n of A is $A - \lambda I = 0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & -\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (8-\lambda) \left[(7-\lambda)(3-\lambda) - 16\right] + 6\left[-6(3-\lambda) + 8\right] + 2\left[24 - 2(7-\lambda)\right] = 0$$

$$\Rightarrow (8-\lambda)\left[\lambda^2 - 10\lambda + 5\right] + 6\left[6\lambda - 10\right] + 2\left[2\lambda + 10\right]$$

$$= 8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 4\lambda + 20 = 0.$$

$$= -\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$= -\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$= \lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$= \lambda(\lambda - 3)(\lambda - 15) = 0$$

$$\lambda = 0, 3, 15.$$

1. Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a characteristic vector corresponding to $\lambda = 0$

Then $Ax = \lambda x \Rightarrow Ax = 0X = 0$

$$= \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow{R_1 \cdot 3} \begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\xrightarrow[R_3 - 4R_1]{R_2 + 3R_1} \begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\xrightarrow[R_3 \times 1/10]{R_2 \times 1/5} \begin{bmatrix} 2 & -4 & 3 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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$\xrightarrow{R_3+R_2}$ $\begin{bmatrix} 2 & -4 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

The above Equation are $2x_1 - 4x_2 + 3x_3 = 0$ & $-x_2 + x_3 = 0$

if $x_2 = k$ then $x_3 = k$

$\therefore 2x_1 = 4k - 3k = k$

$\therefore x_1 = k/2$

Hence the characteristic vectors corresponding to $\lambda = 0$ are $k \begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$

where k is a non-zero scalar

2.) let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a characteristic vector corresponding to $\lambda = 3$ then $AX = 3X$

$\Rightarrow (A - 3I)X = 0$

$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow{R_1 3} \begin{bmatrix} 2 & -4 & 0 \\ -6 & 4 & -4 \\ 5 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\xrightarrow{R_1 \times 1/2}$ $\begin{bmatrix} 1 & -2 & 0 \\ -3 & 2 & -2 \\ 5 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ $\xrightarrow[R_3 - 5R_1]{R_2 + 3R_1}$

$R_2 \times 1/2$

$\begin{bmatrix} 1 & -2 & 0 \\ 0 & -4 & -2 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow{R_1 \times 3} \begin{bmatrix} 1 & -2 & 0 \\ 0 & -4 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$$R_1 \times -\tfrac{1}{2} \rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Hence the equations are $x_1 - 2x_2 = 0$.

$2x_2 + x_3 = 0$

Hence the characterstic vectors corresponding to $\lambda = 3$ are $K \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ where K is a non-zero Scalar.

Let $\lambda = 15$ & $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a characteristic vector corresponding to $\lambda = 15$.

$\therefore AX = 15X$

$\Rightarrow [A - 15I]X = 0$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow{R_{1,3}}$$

$$\begin{bmatrix} 2 & -4 & -12 \\ -6 & -8 & -4 \\ -7 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow[R_2 \times \frac{1}{2}]{R_1 \times \frac{1}{2}} \begin{bmatrix} 1 & -2 & -6 \\ -3 & -4 & -2 \\ -7 & -6 & 2 \end{bmatrix}$$

$$\xrightarrow[R_3 + 7 \cdot R_1]{R_2 + 3 \cdot R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & -10 & -20 \\ 0 & -20 & -40 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \xrightarrow[R_3 \times -\frac{1}{20}]{R_2 \times -\frac{1}{10}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0 \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

Hence the Equation are $x_1-2x_2-6x_3=0$

$x_2+2x_3=0$

if $x_3=k$ then $x_2=-2k$

$x_1=-4k+6k=2k$

Hence the Characteristic vectors Corresponding to $\lambda=15$ are $k=\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ where k is a non-zero scalar.

Conclusion:-

The Characteristic vectors of A are 0, 3, 15.

1. The Charactistic vectors corresponding to $\lambda=0$ are $k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ where k is a non zero scalar.

2. The characteristic vectors corresponding to $\lambda=3$ are $k\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ where k is a nonzero Scalar.

3. The characteristic vectors Corresponding to $\lambda=15$ are $k\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ where k is a non-zero scalar.

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Explain Bessel's InEquality

Date 22-02-2022

Expt. No. MECS

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State and prove Bessel's inequality in an inner product space.

Statement:- If $S=\{\alpha_1, \alpha_2 ---- \alpha_n\}$ is an orthonormal subset in an inner product space $V(f)$ and $\beta \in V$ then

$$\sum_{i=1}^{n} |<\beta, \alpha_i>|^2 \leq \|\beta\|^2$$

Proof:- $S=\{\alpha_1, \alpha_2, ---- \alpha_n\}$ is an orthonormal set.

$$\Rightarrow <\alpha_i, \alpha_j> = \begin{Bmatrix} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{Bmatrix}$$

Consider, the vector $\gamma \in V$ such that $\gamma = \beta - \sum_{i=1}^{n} <\beta, \alpha_i>\alpha_j$

$$<\gamma, \gamma> = <\beta - \sum_{i=1}^{n} <\beta \alpha_i> \alpha_i, \beta = \sum_{i=1}^{n} <\beta, \alpha_i> \alpha_i$$

$$= (\beta, \beta) = <\beta, \sum_{i=1}^{n} <\beta, \alpha_i>\alpha_i> - <\sum_{i=1}^{n} <\beta, \alpha_i>\alpha_i, \beta>$$

$$+ <\sum_{i=1}^{n} <\beta, \alpha_i>\alpha_i, \sum_{i=1}^{n} <\beta, \alpha_i>\alpha_i>$$

$$= \|\beta\|^2 - \sum_{i=1}^{n} <\bar{\beta}, \alpha_i><\beta, \alpha_i> - \sum_{i=1}^{n} <\beta, \alpha_i><\alpha_i, \beta>$$

$$+ \sum_{i=1}^{n} <\beta, \alpha_i> \sum_{i=1}^{n} <\overline{\beta, \alpha_i}><\alpha_i, \alpha_i>$$

$$= \|\beta\|^2 - \sum_{i=1}^{n} <\beta, \overline{\alpha_i}><\beta, \alpha_i> - \sum_{i=1}^{n} <\beta, \alpha_i><\beta, \alpha_i>$$

$$+ \sum_{i=1}^{n} (\beta, \alpha_i) <\beta, \alpha_i><\alpha_i, \alpha_i>$$

$$= \|\beta\|^2 - \sum_{i=1}^{n} <\bar{\beta}, \alpha_i><\beta, \alpha_i>$$

Signature:

$\|\beta\|^2 - \sum_{i=1}^{n} \|z\|^2$

$<\gamma, \gamma> = \|\beta\|^2 - \sum_{i=1}^{n} |<\beta, \alpha_i>|^2$

we know that $\|\gamma\|^2 \geq 0$

$\Rightarrow \|\gamma\|^2 \geq 0$

$\Rightarrow <\gamma, \gamma> \geq 0$

$\Rightarrow \|\beta\|^2 - \sum_{i=1}^{n} |<\beta, \alpha_i>|^2$

$\Rightarrow \sum_{i=1}^{n} |<\beta, \alpha_i>|^2 \leq \|\beta\|^2$

9. State and prove Stoke's Theorm

Statement: let "s" be a closed surface enclosed by a curve "c" If "f" continuously differentiable vector point function, then

$$\boxed{\int_S \text{Curl } f \cdot N ds = \oint_C f \cdot dr}$$

Proof:- let "s" be a closed surface enclosed by curve "c"

$$F = F_1 i + F_2 j + F_3 k$$

Consider $\nabla \times F_1 i = \begin{vmatrix} i & j & k \\ \delta/dx & \delta/dy & \delta/dz \\ f_1 & 0 & 0 \end{vmatrix}$

$$= i(0-0) - j\left(0 - \frac{\delta f_1}{\delta z}\right) + k\left(0 - \frac{\delta f_1}{dy}\right)$$

$$= \frac{\delta f_1}{\delta z} \cdot j \cdot N ds - \frac{\delta f_1}{\delta y_1} k \cdot N ds \quad \text{——} (1)$$

let $z = f(x,y)$ be the equation of S

for any point on S, $r = xi + yj + zk$

$$\Rightarrow \frac{dr}{\delta y} = j + \frac{\delta z}{dy} \cdot k$$

Let $z = f(x,y)$ be the equation of S

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Since $\frac{dr}{dy}$ is a tangent vector to S and N is normal to S.

$$\frac{dr}{dy} \cdot N = 0$$

$$\left(j + \frac{dz}{dy} \cdot k\right) N = 0$$

$$\left(j + \frac{dz}{dy} \cdot k\right) N = 0$$

$$j \cdot N + \frac{dz}{dy} (k \cdot N) = 0$$

$$J \cdot N = -\frac{dz}{dy} (k \cdot N)$$

Substituting this value in Eq^n (1)

$$(\nabla \times f_1 i) N ds = \frac{df_1}{dz}\left[-\frac{dz}{dy}(k \cdot N)\right] \cdot ds - \frac{df_1}{dy} k \cdot N ds$$

$$= -\left[\frac{df_1}{dz} \cdot \frac{dz}{dy} + \frac{df_1}{dy}\right] k \cdot N ds.$$

$$= -\frac{\delta f_1}{dy} k \cdot N ds$$

Let R be the projection of S in xy plane

$$ds = \frac{dx \cdot dy}{N \cdot k}$$

$$N \cdot k \, ds = \delta x \cdot \delta y$$

Now. $\int (\nabla \times f_1 i) N ds = -\frac{\delta f_1}{dy} k \cdot N ds$

$$= \iint_S -\frac{df_1}{dy} dx \cdot dy$$

$$= \oint_c f_1 \, dx \quad (\text{By greens theorm})$$

$\because \int (\nabla \times f_1 i) \cdot N ds = \oint_C f_1 \cdot dx \rightarrow$ A

Similarly $\int_S (\nabla \times f_2 j) \cdot N ds = \oint_C f_2 \cdot dy \rightarrow$ B

$\int_S (\nabla \times f_3 k) \cdot N ds = \oint_C f_3 \cdot dz \rightarrow$ C

A + B + C

$$\int_S \nabla \times (f_1 i + f_2 j + f_3 k) \cdot N ds = \oint_C f_1 dx + f_2 dy + f_3 dz$$

$$\Rightarrow \int_S (\nabla \times f) N ds = \oint_C f \cdot dr$$

$$\boxed{\int_S \text{curl } f = \oint_C f \cdot dr}$$

This is called "stoke's theorm"

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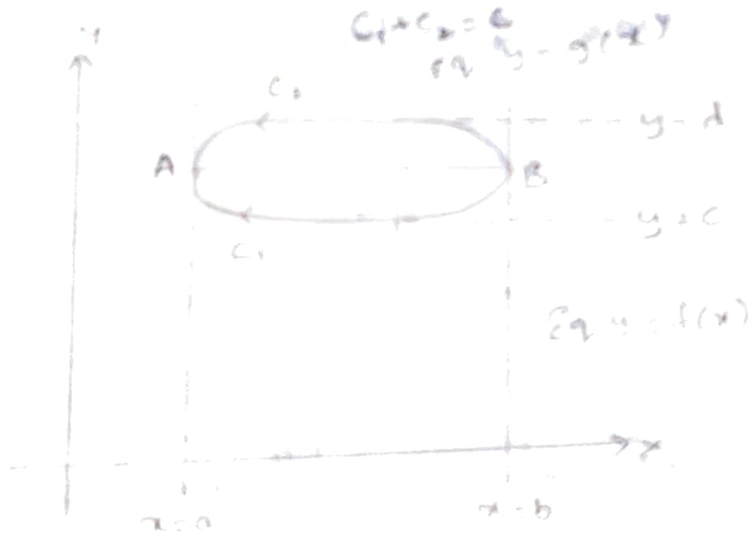
10. State and prove Green's Theorem

<u>Statement</u> :- If P and Q are two continuously differentiable scalar point functions, then.

$$\oint_C P\,dx + Q\,dy = \iint_S \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy$$

<u>Proof</u> :- let P and Q be two continuously differentiable scalar point function and "S" be closed surface enclosed by a curve "C".



let C_1 be the lower point of C and C_2 be the upper point of C.

let $y=f(x)$ be the equation of C_1 and $y=g(x)$ be the equation of C_2

Suppose "S" lies b/w $x=a, y=c, x=b, y=0$

Now $\iint_S \frac{dP}{dy}\,dx\,dy = \int_{x=a}^{x=b}\left[\int_{y=f(x)}^{y=g(x)} \frac{dP}{dy}\,dy\right]dx$

$$= \int_{x=a}^{x=b} \left[P(x,y) \right]_{y=f(x)}^{y=g(x)} dx$$

$$= \int_{x=a}^{y=b} \left[P(x), g(x) - P(x), f(x) \right] dx$$

$$= \int_{x=a}^{x=b} P(x), g(x)\, dx - \int_{x=a}^{x=b} P(x), f(x)\, dx$$

$$= - \left[\int_{C_1} + \int_{C_2} P(x,y)\, dx \right]$$

$$= -\oint_C P dx$$

$$\therefore \iint_C \frac{dP}{dy} dx\, dy = \oint_C P . dx$$

$$\Rightarrow \oint_C P dx = \iint_C \frac{dP}{dy} dx . dy \leftarrow A$$

$$\text{Illy} \quad \oint_C Q dy = \iint_S \frac{dQ}{dx} dx . dy \rightarrow B$$

A + B

$$\oint_C P dx + Q dy = -\iint_S \frac{dP}{dy} dx . dy + \iint_S \frac{dQ}{dx} dx . dy$$

$$\oint P dx + Q dy = \iint_S \left(\frac{dQ}{dx} + \frac{dP}{dy} \right) dx . dy$$

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$\therefore \oint_C P\,dx + Q\,dy = \iint_S \left(\frac{dQ}{dx} + \frac{dP}{dy}\right) dx \cdot dy$

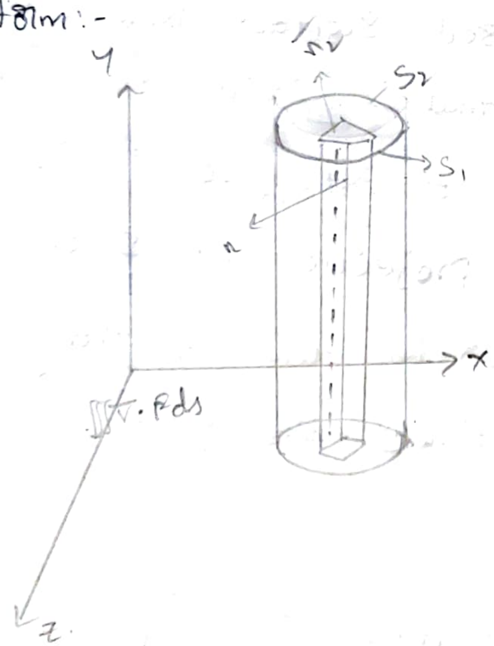
$\therefore$ This is called green's theorem.

T. Mohani kumari

1. State and prove Gauss Divergence Theorem.

Statement:- If S is a closed Surface enclosed a volume v and f is a continuously differentiable vector point function then $\int_v \mathrm{div}.\, f dv = \int_S F.N ds$ where N is a unit outward drawn Normal at any point on S.

Cartesian form:-



Let $F = F_1 i + F_2 j + F_3 k$. Let the unit normal vector N_1 drawn outward make angles α, β, γ with positive direction of the coordinate axes then

$N = \cos\alpha i + \cos\beta j + \cos\gamma k$

$F.N = F_1 \cos\alpha + F_2 \cos\beta + F_3 \cos\gamma$ and

$$\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Divergence theorm in Cartesian form. Can be written as

$$\iiint_V \left[\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right] dx\,dy\,dz = \iint_S (F_1 \cos\alpha + F_2 \cos\beta + F_3 \cos\gamma)\,ds$$

$$= \iint_S (f_1\,dy\,dz + f_2\,dz\,dx + f_3\,dx\,dy)$$

Proof :-

Let S be a closed surface. Now choose the coordinate axes such that any line drawn parallel to the axes cuts S in atmost two points.

Let R be the projection of S on xy-plane.

Let $z = f(x,y)$ and $z = g(x,y)$ be the equations of S_1 and S_2 which are respectively the lower and upper parts of S.

$$\text{Now } \int_V \frac{\partial f_3}{\partial z}\,dv = \iiint_V \frac{\partial f_3}{\partial z}\,dx\,dy\,dz = \iint_R F_3(x,y,z)\Big|_{z=f(x,y)}^{z=g(x,y)} dx\,dy$$

$$= \iint_R [F_3(x,y,g) - f_3(x,y,f)]\,dx\,dy$$

$$= \iint_R [R_3(x,y,g)]\,dx\,dy - \iint_R [F_3(x,y,f)]\,dx\,dy \quad -(1)$$

Subhodaya

for the upper part S_2 of S, $dx\,dy = ds \cdot \cos\gamma = N \cdot k\,ds$

$$\iint_R f_3 (x, y, z)\, dx\, dy = \int_{S_1} f_3 \cdot N \cdot k\, ds$$

for the lower part S_1 of S, $dx\,dy = -ds \cdot \cos\gamma = -N \cdot k\,ds$

$$\iint_R f_3 (x, y, f)\, dx\, dy = -\int_{S_2} F_3\, N \cdot k\, ds$$

Hence from (1) $\displaystyle\int_V \frac{\partial F_3}{\partial z}\, dv = \int_{S_2} F_3 N \cdot k\,ds = \int_S F_3\, N \cdot k\,ds$

Illy by projecting S on the other coordinate planes

we get

$$\int_V \frac{\partial f_2}{\partial y}\, dv = \int_S F_2 N \cdot j\, ds \quad \text{—(2)}$$

and

$$\int_V \frac{\partial F_1}{\partial x}\, dv = \int_S F_1 N \cdot i\, ds \quad \text{—(3)}$$

Adding the eqns (1), (2) and (3)

$$\int_V \left[\frac{\partial f_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right] dv = \int (F_1 i + F_2 j + F_3 k) \cdot N\, ds$$

$\therefore$ Hence $\displaystyle\int_V \operatorname{div} F\, dv = \int_S F \cdot N\, ds$

$\therefore$ This is called Gauss Divergence theorm.

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24) L. Himabindu	MECs	1900140023 (44)
25) D. yamuna	MECs	1900140014 (33)
26) S. Joshnarani	MECs	1900140048 (43)
27) A. Yellamma	MECS	1900140003 (41)
28) S. Harathi	MPcs	1900142045 (71)
29) P. Manoj	Mpcs	1900142040 (46)
30) R. Ammannaidu	Mpcs	1900142044 (49)
31) L. Krishna Rao	MECS	1900140024 (09)
32) P. Anil Kumar	MECS	1900140043 (28)
33) L. Chindu	mecs	1900140022 (36)
34) T. Kusuma	MEC'S	1900140051 (11)
35) T. Mohini kumari	MPC'S	1900142054 (60)
36) K. Ananth Rao	MPE	1900143020 (25)
37) T. Priyanka	MECS	1900140049 [51]
38) B. Manoj	mpcs	1900142011 (13)
39) B. Jawahar babu	MPCS	1900142010 (12)
40) D. Srinivasarao	M.P.C	1900141008 (10)
41) Ch. Bhankararao	M.P.C	1900141006 (8)
42) I. Mohan Rao.	MPCS	1900142021 (23)
43) Judisti Baharo[illegible]	MPcs	[illegible]

No.	Name	Group	Hall ticket N
44)	A. chinna babu	M.P.CS	1900142001 (
45)	B. Eswara Rao	M.C.I.C	1900138001 (0
46)	S. Lakshmana	M.P.C	190014104
47)	T. Srinivas	MECS	1900140050 (22)
48)	D. Vinay	MPCS	1900142015 (17
49)	K. Ramesh kumar	MPCS	1900142026 (27
50)	Nirmal pradhan	MPCS	1900142034 (38
51)	ERIPILLI. DILEEP	MPC	1900141014 (16)
52)	UPPADA. NAVEEN	MPC	1900141047 (58)

Add

51)	P. Raju	MPCS	1800142037 (50)
52)	Srikakulam. pavanchandu	MECS	1800140041 (20)
53)	BORA. SATHESH	M.P.C	1900141004 (6).
54)	K. srikanth	M.P.CS	1900142028 (31)
55)	T. Gayatri	M.P.CS	1900142053 (59)

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